Technical Comments

Comment on "Effect of Nonlinear Prebuckling State on the Postbuckling Behavior and Imperfection-Sensitivity of Elastic Structures"

J. R. FITCH*
Knolls Atomic Power Laboratory, General Electric Company, Schenectady, N.Y.
AND
J. W. HUTCHINSON†
Harvard University, Cambridge, Mass.

COHEN[1] has presented a general development of an initial postbuckling analysis which takes into account nonlinear prebuckling deformations of an elastic structure. He reports a discrepancy between several of his results and those obtained in a similar analysis by Fitch.[2] As briefly as possible, we will resketch this analysis, following the same notation as that used by Cohen, to show that Fitch's results are the correct ones. The reader is referred to these two papers for definitions and details which will not be repeated here.

The discrepancy between the two sets of results arises in the analysis of the effect of a small imperfection on the buckling load of the structure. If the imperfection is denoted by $\bar{\xi}\bar{u}$ where $\bar{\xi}$ is the imperfection amplitude, then the strain resulting from an additional displacement u is

$$\epsilon = L_1(u) + \tfrac{1}{2}L_2(u) + \bar{\xi}L_{11}(u,\bar{u}) \qquad (1)$$

As Cohen asserts, the displacement can be written as

$$u = u_0 + \xi u_1 + \xi^2 u_2 + \xi^3 u_3 + \ldots \qquad (2)$$

since to the order to which the analysis is carried, terms such as $\xi\bar{\xi}u_{11}$, etc. do not enter. Thus, the strain can be written as

$$\epsilon = \epsilon_0 + \bar{\xi}L_{11}(\bar{u},u_0) + \xi[L_1(u_1) + L_{11}(u_0,u_1)] + \xi^2[L_1(u_2) + \tfrac{1}{2}L_2(u_1) + L_{11}(u_0,u_2)] + \xi^3[\quad] + \xi\bar{\xi}[\quad] + \ldots \qquad (3)$$

where $\epsilon_0 \equiv L_1(u_0) + \frac{1}{2}L_2(u_0)$ is the strain arising from the prebuckling deformation of the perfect structure u_0. The variation in the strain can also be written as

$$\delta\epsilon = L_1(\delta u) + L_{11}(u_0,\delta u) + \bar{\xi}L_{11}(\bar{u},\delta u) + \xi L_{11}(u_1,\delta u) + \xi^2 L_{11}(u_2,\delta u) + \ldots \qquad (4)$$

Now, these expansions are substituted into the principle of virtual work (here we will consider only dead loading)

$$\sigma\cdot\delta\epsilon = q\cdot\delta u \qquad (5)$$

Here the stress-strain relation $\sigma = H(\epsilon)$ is used along with the fact that the prebuckling deformation, itself, satisfies

$$\sigma_0\cdot\delta\epsilon_0 = q\cdot\delta u$$

where $\delta\epsilon_0 \equiv L_1(\delta u) + L_{11}(u_0,\delta u)$. A nonlinear algebraic equation relating ξ, $\bar{\xi}$, and the load parameter is obtained by taking $\delta u = u_1$ and by using the equations for u_1, u_2, etc. listed by Fitch and Cohen. This is similar to what has been done by Cohen[1]; but the term $\bar{\xi}L_{11}(\bar{u},u_0)$ in the expression for ϵ seems to have been omitted in that reference. Thus, for example, the term given on p. 1619 of Ref. 1 for augmenting the right-hand side of Eq. (6c) is incorrect.

The final results obtained are in the form of Cohen's Eqs. (33 and 35–38) but with corrected expressions for α and β. In particular, we find

$$\alpha = [\sigma_0{}^*\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^*)]/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

with a similar type of expression, if the live loading term q_1 is taken into account. When the imperfection is in the shape of the buckling mode (i.e., $\bar{u} = u_1$), this reduces to

$$\alpha = [\sigma_0{}^*\cdot L_2(u_1) + \sigma_1\cdot L_{11}(u_1,u_0{}^*)]/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

or, alternatively, this can be written in the form equivalent to that given by Fitch,

$$\alpha = -\sigma_1\cdot L_1(u_1)/[\lambda_C\sigma_0{}^{(1)*}\cdot L_2(u_1) + 2\lambda_C\sigma_1\cdot L_{11}(u_0{}^{(1)*},u_1)]$$

The discussion given at the end of Ref. 1 concerning the effect of imperfections other than those in the shape of the buckling mode must be changed accordingly to account for the correct expression for α.

References

[1] Cohen, G. A., "Effect of Nonlinear Prebuckling State on the Postbuckling Behavior and Imperfection-Sensitivity of Elastic Structures," *AIAA Journal*, Vol. 6, No. 8, Aug. 1968, pp. 1616–1619.

[2] Fitch, J. R., "The Buckling and Post-Buckling Behavior of Spherical Caps under Concentrated Load," *International Journal of Solids Structures*, Vol. 4, 1968, pp. 421–446.

Received September 1968.

* Applied Mathematics Group.

† Professor of Applied Mechanics, Division of Engineering and Applied Physics.

Reply by Author to J. R. Fitch and J. W. Hutchinson

GERALD A. COHEN*
Structures Research Associates, Newport Beach, Calif.

THE author would like to thank J. R. Fitch and J. W. Hutchinson for correcting the error in the subject paper. The error arose as a result of applying the third of Eqs. (5) to the imperfect structure, whereas only the first of Eqs. (5) remains valid for this case.

As a result of this correction, the following equations of the subject paper should be changed as shown:

$$\alpha = [\sigma_0{}^*\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^*) - \lambda_c q_1(\bar{u})\cdot u_1]/\lambda_c F^{(1)}(u_1,u_1) \qquad (34a)$$

$$\beta = \{\sigma_0{}^{(1)*}\cdot L_{11}(\bar{u},u_1) + \sigma_1\cdot L_{11}(\bar{u},u_0{}^{(1)*}) - q_1(\bar{u})\cdot u_1 + H[L_{11}(u_0{}^{(1)*},u_1)]\cdot L_{11}(\bar{u},u_0{}^*) - \alpha\lambda_c H[L_{11}(u_0{}^{(1)*},u_1)]\cdot L_{11}(u_0{}^{(1)*},u_1) - \tfrac{1}{2}\alpha\lambda_c F^{(2)}(u_1,u_1)\}/F^{(1)}(u_1,u_1) \qquad (34b)$$

Received March 24, 1969.

* President and Technical Director. Member AIAA.

$$\alpha = -\sigma_1 \cdot L_1(u_1)/\lambda_c F^{(1)}(u_1,u_1) \quad (\bar{u} = u_1) \tag{39}$$

$$X = T_1^{(0)*}\chi^{(1)} + T_{12}^{(0)*}\psi^{(1)} + T_1^{(1)}\chi^{(0)*} + T_{12}^{(1)}\psi^{(0)*} \tag{42a}$$

$$Y = T_2^{(0)*}\psi^{(1)} + T_{12}^{(0)*}\chi^{(1)} + T_2^{(1)}\psi^{(0)*} + T_{12}^{(1)}\chi^{(0)*} \tag{42b}$$

$$\alpha = \mu/\lambda_c F^{(1)}(u_1,u_1) \tag{45}$$

With these corrections, the discussion of general imperfections given at the end of the paper remains valid. Equations (44) then give an imperfection shape proportional to the buckling mode only in the special case of a prebuckling state characterized by equal constant normal stress resultants and identically zero shear-stress resultant and rotations.

As a further observation, it is noted that the equations giving the postbuckling coefficients $a, b, \ldots$ [Eqs. (22) of the subject paper] are, in fact, the compatibility conditions for the equations giving the functions u_2, u_3, ..., respectively. Thus, the equation for a is obtained by setting $\delta u = u_1$ in Eq. (13c) and $\delta u = u_2$ in Eq. (8c), and subtracting the results. Similarly, b is obtained by setting $\delta u = u_1$ in Eq. (14c) and $\delta u = u_3$ in Eq. (8c) and subtracting the results. This derivation of the postbuckling coefficients is more direct than the derivation given in the paper.

Comments on "Acceleration Process in a Stabilized High-Current Arc"

F. Y. Sorrell*
North Carolina State University, Raleigh, N.C.

In a recent Note, Burton and Chang[1] report on current sheet dynamics in a pulsed parallel plate accelerator with and without electrode insulation. In this paper, they state that previous work did not investigate the effect of electrode insulation on acceleration process, but the author[2] has reported similar experiments in an inverse pinch with and without insulated electrodes. This is difficult to understand because Ref. 2 was also included as a reference in the paper by Burton and Chang.[1]

In view of their overlooking this work with insulated electrodes, the author would like to compare the work of Burton and Chang[1] with that reported in Ref. 2. In these experiments in an inverse pinch, the electrode was a tungsten button approximately one inch in diameter with pyrex or quartz disks as the "insulating electrodes." Current sheet behavior in this system is described in detail in Ref. 3, but a very brief summary is given here emphasizing the aspects that were significantly different from those observed by Burton and Chang.[1]

Received January 22, 1969.

* Assistant Professor, Department of Engineering Mechanics.

In the experiments in the inverse pinch,[3] the current sheet was found to be completely stable and to propagate in a relatively thin uniform sheet. There was very little tilt of the current sheet and no diffuse areas near the electrodes were observed. In fact, insulating electrodes were used specifically to remove the usual diffuse region near the electrode and current sheet tilt. The reasons that the insulating electrodes achieve this are discussed in Ref. 2. In addition, under no conditions were the propagation of current vortices, reported in Ref. 1, observed.

As discussed by Burton and Chang,[1] there is good reason to believe that ablated material from the electrodes and insulators greatly affect the current sheet pattern. The work in the inverse pinch was primarily a study of the production of strong shock waves and thus great care was used to achieve a clean experiment. This is evidenced by the use of aluminum oxide, pyrex, and quartz as insulating materials, and tungsten as electrodes in the entire experiment. However, in Ref. 1 the insulator was nylon and the electrodes were brass, both of which can yield a much greater amount of impurities. It is the author's belief that the possibility of increased impurities in the experiments of Burton and Chang[1] could have produced the difference in data observed between their experiments and those reported in Ref. 2.

References

[1] Burton, R. L. and Chang, O. Y., "Acceleration Process in a Stabilized High-Current Arc," *AIAA Journal*, Vol. 6, No. 11, Nov. 1968, pp. 2190–2192.

[2] Sorrell, F. Y., Ph.D. thesis, 1966, California Institute of Technology, Pasadena, Calif.

[3] Sorrell, F. Y., "Current Sheet Dynamics in an Inverse Pinch," *The Physics of Fluids*, Vol. 11, No. 5, May 1968, pp. 993–1001.

Reply by Authors to F. Y. Sorrell

Rodney L. Burton* and O. Y. Chang†
University of California, San Diego, Calif.

We are fully aware that F. Y. Sorrell has performed and reported experiments in an inverse pinch with insulated electrodes, and did not intend to give the impression that his work was overlooked. We are not sure why the current sheet in his insulated inverse pinch propagates in snowplow fashion, whereas the current sheet in our insulated parallel plate has a zero velocity and a grossly steady magnetic field pattern. Sorrell comments on the possibility of impurities in our experiment. Indeed, as mentioned in the Note, it is our belief that our arc runs entirely on vaporized electrode material.

Received February 28, 1969.

* Assistant Professor of Engineering Physics, Department of Aerospace and Mechanical Engineering Sciences. Member AIAA.

† Research Assistant, Department of Aerospace and Mechanical Engineering Sciences.